

# Multinomial and ordered logits

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# GLM: A recap

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⇒ *Probability distribution maps unobserved variable to observed outcomes.*

# Ordered logistic regression



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⇒ *estimate a single set of regression parameters, but keep the information on the order without assuming a continuous variable.*

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## Latent variable approach: cutpoints

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- ▶ **Ordinal logistic:** Several cutpoints. → Explicit.

⇒ *Model estimates both regression parameters ( $\beta$ ) and cutpoints ( $\tau$ ).*

## A series of cutpoints

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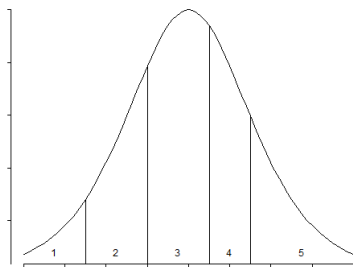


Figure 1: Slicing up a latent variable

# The regression coefficients

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- ▶ You end up with  $m - 1$  cutpoints.

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## The predicted value

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$$Pr(y_i = m) = \frac{\exp(\tau_m - \beta x_i)}{1 + \exp(\tau_m - \beta x_i)} - \frac{\exp(\tau_{m-1} - \beta x_i)}{1 + \exp(\tau_{m-1} - \beta x_i)} \quad (1)$$

## An example: Attitudes towards redistribution

## An example:

ESS respondents (that voted V or DF) are asked to what extent they believe the state should engage in redistribution (1 = disagree; 5 = agree).

```
#Load in data
download.file(
  url("https://siljehermansen.github.io/teaching/beyond-linear-models/kap10.rda")
  destfile = "kap10.rda"
)
df <- kap10

#Check distribution
barplot(table(df$Udjaevn))
```

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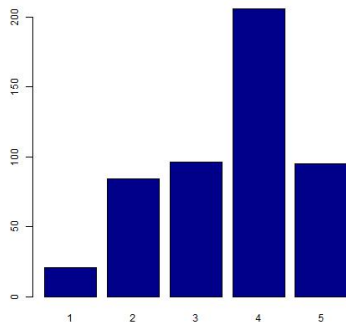


Figure 2: Attitudes towards redistribution is an ordered variable

# Attitudes towards redistribution as a function of income

```
#Library for ordinal regression
library(MASS)
#Recode into ordered factor
df$Udjaevn.ord <- as.ordered(as.factor(df$Udjaevn))
#Run regression
mod.ord <- polr(Udjaevn.ord ~ Indtaegt,
                df,
                method = "logistic",
                Hess = TRUE)
summary(mod.ord)
```

# Attitudes towards redistribution as a function of income

```
## Call:
## polr(formula = Udjaevn.ord ~ Indtaegt, data = df, Hess = TRUE,
##       method = "logistic")
##
## Coefficients:
##           Value Std. Error t value
## Indtaegt 0.1153    0.03155   3.653
##
## Intercepts:
##      Value  Std. Error t value
## 1|2 -2.4186   0.2903   -8.3306
## 2|3 -0.6008   0.2179   -2.7566
## 3|4  0.3069   0.2150    1.4277
## 4|5  2.2276   0.2403    9.2686
##
## Residual Deviance: 1298.396
## AIC: 1308.396
## (51 observations deleted due to missingness)
```

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⇒ *Hypothesis testing as in a binomial logit*

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- ▶ e.g.: intercept is reported as significant (with standard errors)

⇒ *The model does a fair job in distinguishing between categories.*

## Predicted scenarios

**We interpret predicted probability by choosing one level of  $x$  and one category (two cutpoints) of  $y$ : What is the probability of  $m$ ?**

$$Pr(y_i = m) = \frac{\exp(\tau_m - \beta x_i)}{1 + \exp(\tau_m - \beta x_i)} - \frac{\exp(\tau_{m-1} - \beta x_i)}{1 + \exp(\tau_{m-1} - \beta x_i)} \quad (2)$$



## Example

Let's choose low-income respondents ( $x = 1$ ) and category 3 (diff between cutpoints 2 and 3)

```
z = mod.ord$zeta
x = 1

logodds1 <- z[3] - coefficients(mod.ord) * x
logodds2 <- z[3-1] - coefficients(mod.ord) * x
## Probabilities
p1 <- exp(logodds1)/(1 + exp(logodds1)) #3/4 or lower
p2 <- exp(logodds2)/(1 + exp(logodds2)) #2/3 or lower
## Difference between cutpoints
p1 - p2 #cat 3
```

## An example

### Predicted proportion in category

```
paste(round((p1-p2)*100),  
"% of low-income respondents are predicted to answer x = 3 ('neutral')." )
```

[1] "22 % of low-income respondents are predicted to answer  $x = 3$  ('neutral')."

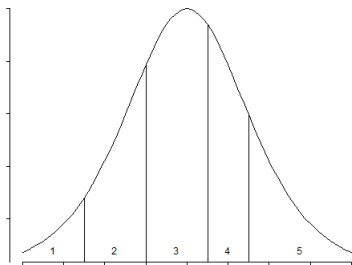
### Cumulative probability

```
paste(round((p1)*100),  
"% of low-income respondents are predicted to answer x = 3 ('neutral') or lower")
```

[1] "55 % of low-income respondents are predicted to answer  $x = 3$  ('neutral') or lower to the question of whether they support redistribution."

## Two ways of viewing the slicing

We can report the probability (e.g. 0.22) of ending up between two cutpoints, or the *cumulative* probability (e.g. 0.55) to be below each



point.

## Exercise:

Increase the  $\tau$  ( $z$ ) within each value of Income ( $x$ )

```
##Create empty plot
plot(y = 0,
     x = 0,
     axes = FALSE,
     xlim = c(1,4),
     ylim = c(0,1),
     ylab = "Probability of z or below",
     xlab = "Thresholds",
     main = "Cumulative probability \nof support for redistribution",
     type = "n")
axis(1, at = 1:length(p1),
     labels = names(p1))
axis(2)
```

## Exercise:

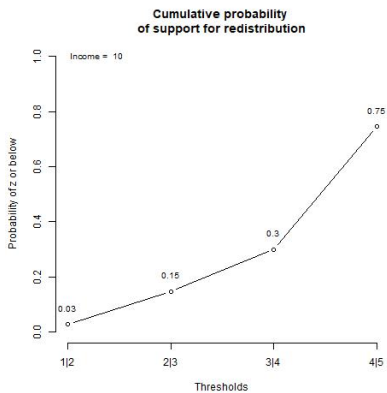
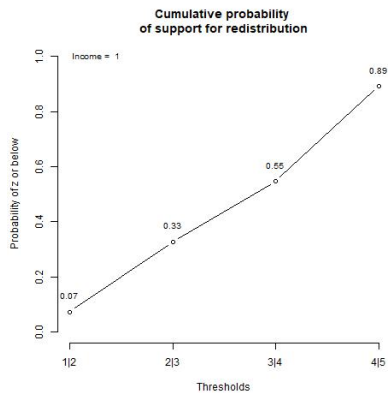
Increase the  $\tau$  ( $z$ ) within each value of Income ( $x$ )

```
#Set values for prediction
x = 10 #Let this go from 1 to 10; check the shape of 10
z = mod.ord$zeta
#Logodds
logodds1 <- z - coefficients(mod.ord) * x
#Probabilities
p1 <- exp(logodds1)/(1 + exp(logodds1)) #3/4 or lower

#Plot probabilities
lines(y = p1,
      x = 1:length(p1),
      type = "b")
#Set legend (report x-value)
legend("topleft",
      bty = "n",
      cex = 0.8,
      paste("Income = ", x))

#Plot probabilities
text(x = 0, y = 0.15, label = "0.15")
```

# Result



## Parallel regressions approach: for assessment

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- ▶ Run a series of regressions where all  $\beta$  are fixed (i.e.: the same).

$\Rightarrow$  *This is also useful when we assess the model*

How good is our model?

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```
df %>%
  filter(!is.na(Udjaevn)) %>%
  group_by(Udjaevn) %>%
  summarize(mean(Indtaegt, na.rm = T))
```

```
## # A tibble: 5 x 2
##   Udjaevn 'mean(Indtaegt, na.rm = T)'
##   <dbl>          <dbl>
## 1         1         4.8
## 2         2         5.58
## 3         3         5.96
## 4         4         6.41
## 5         5         6.75
```

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```

- ▶ Run parallel regressions without constraint on  $\beta$ . Are they similar?



## An example of parallel regressions

## Recode into dummies

The dummies flag cases below a cumulative threshold of *outcomes*

```
##  
df$ut1 <- ifelse(df$Udjaevn > 1, 1, 0) #2 or above  
df$ut2 <- ifelse(df$Udjaevn > 2, 1, 0) #3 or above  
df$ut3 <- ifelse(df$Udjaevn > 3, 1, 0) #4 or above  
df$ut4 <- ifelse(df$Udjaevn > 4, 1, 0) #5
```

⇒ The model then runs 4 regressions where  $\beta$  reports an aggregated value from all 4 coefficients (think: weighted mean).

## Run four regressions

Let's exemplify with the parallel regressions without fixed  $\beta$ :

```
##Parallel regressions:
```

```
mod1 <- glm(ut1 ~ Indtaegt, df, family = "binomial")  
mod2 <- glm(ut2 ~ Indtaegt, df, family = "binomial")  
mod3 <- glm(ut3 ~ Indtaegt, df, family = "binomial")  
mod4 <- glm(ut4 ~ Indtaegt, df, family = "binomial")
```

## Compare coefficients from four regressions

```
##
## =====
##                               Dependent variable:
##                               -----
##                               ut1      ut2      ut3      ut4
##                               (1)      (2)      (3)      (4)
##                               -----
## Indtaegt      0.189**   0.125***  0.110***  0.094**
##               (0.085)   (0.041)  (0.035)  (0.045)
##
## Constant      2.048***  0.552**   -0.270   -2.082***
##               (0.474)   (0.260)  (0.231)  (0.319)
##
## -----
## Observations      459      459      459      459
## Log Likelihood     -79.653  -234.669 -303.983 -217.674
## Akaike Inf. Crit. 163.306  473.338  611.967  439.348
## =====
## Note:                *p<0.1; **p<0.05; ***p<0.01
```

## Coefficient should be a weighted average from four regressions

These  $\beta$ s are weighted by the number of observations in each category:

```
table(df$Udjaevn)
```

```
##  
##  1  2  3  4  5  
## 21 84 96 206 95
```

We can plot the  $\beta$ s for comparison:

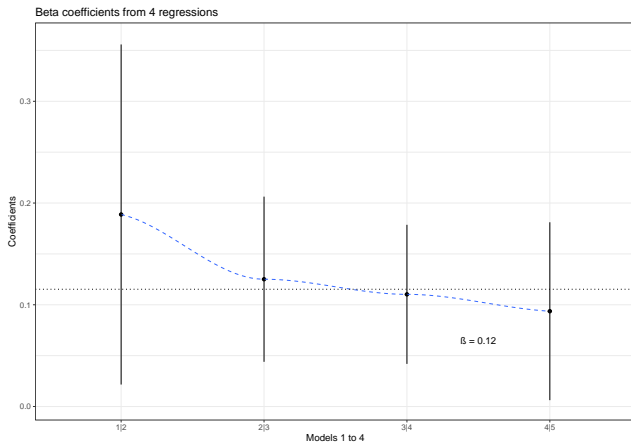
```
results <- rbind(summary(mod1)$coefficients[2, c(1,2)],  
                 summary(mod2)$coefficients[2, c(1,2)],  
                 summary(mod3)$coefficients[2, c(1,2)],  
                 summary(mod4)$coefficients[2, c(1,2)])  
thresholds <- c("1|2", "2|3", "3|4", "4|5")
```

We can plot the  $\beta$ s for comparison:

```
ggplot() +
  geom_point(aes(y = results[, "Estimate"],
                x = thresholds)) +
  geom_smooth(aes(y = results[, "Estimate"],
                 x = 1:4),
             lty = 2,
             lwd = 0.5) +
  geom_segment(aes(x = 1:4,
                  xend = 1:4,
                  y = results[, "Estimate"]-results[, "Std. Error"]*1.96,
                  yend = results[, "Estimate"]+results[, "Std. Error"]*1.96)) +
  theme_bw() +
  ylim(c(results[, "Estimate"][4]-results[, "Std. Error"][4]*2,
         results[, "Estimate"][1]+results[, "Std. Error"][1]*2)) +
  geom_hline(yintercept = mod.ord$coefficients,
            lty = 3) +
  geom_text(aes(y = mod.ord$coefficients-0.05,
               x = 3.5,
               label = paste("\u03b2 =", round(mod.ord$coefficients,2))
               ),
            parse = F) +
  labs(title = "Beta coefficients from 4 regressions") +
  ylab("Coefficients") +
  xlab("Models 1 to 4")
```

We can plot the  $\beta$ s for comparison:

The overall  $\beta$  is 0.12. If the ordered model describes the data well, then all the unconstrained  $\beta$ s should resemble that description.





## A visual inspection

A more visual way of checking the “parallel lines assumption” is to inspect if the regression lines are parallel.

# When is it smart to run an ordered logit?

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- ▶ You have few ordered categories
- ▶ The effect is approximately the same across the categories (parallel lines assumption)

# What do I do if the assumption doesn't hold?

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  - ▶ if you have many categories
  - ▶ fairly equal spread of observations between categories

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- ▶ Run an OLS/linear model:
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- ▶ Run a multinomial model:
  - ▶ i.e. estimate different  $\beta$  for each regression/threshold

# Discrete choice models

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  - ▶ Multinomial: Models *chooser* characteristics
  - ▶ Conditional logit: Models *choice* characteristics

## Multinomial logistic regression

# Two conceptions of multinomial regression

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- ▶ **A series of binomial logits** with the same reference category.
- ▶ **Latent variable approach:** Our utility of each choice.

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**A series of binomial logits** with the same reference category.

- ▶ Data is subset to compare two groups  $\rightarrow$  data/variation intensive model choice.
- ▶ Categories/choice are mutually exclusive  $\rightarrow$  Different  $\beta$  for each subset/choice

$\Rightarrow$  *All choices are given a probability and they sum up to one.*

## Two conceptions of multinomial regression

**Latent variable approach:** Imagine  $k$  choices modeled as  $y_m = \alpha_m \times \beta_m x$

- ▶  $\beta_m x_i$  reflects the utility of a choice  $k$  for the chooser  $i$  with  $x$  characteristic. → systematic term

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⇒ *The preferred choice is the one with the highest utility because both or either are high*



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- ▶ consistency: if we prefer  $A > B$  and  $B > C$ , then also  $A > C$

⇒ *The  $\beta$  does not depend on other values of  $y$  (other alternatives).*

## Testing the main assumption:

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**The Hausmann-McFadden test:** Removes an alternative (supposed to be irrelevant) and check if  $\beta$  changes.

- ▶ Restricted model (a choice is removed) vs. unrestricted model (original)
- ▶ if IIA holds, then unrestricted model has smaller variance.

⇒  $\chi^2$ -test with smaller value indicate IIA holds.

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⇒ *Remember reference category is 1 – the sum of all other probabilities*

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## The conditional logit

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## Mixing choosers and choices

**The mixed conditional logit makes an interaction effect between choice-set variables and choice variables.**

- ▶ Think hierarchical models with cross-level interactions