Count models

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18-11-2019

The dependent variable

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Count models: What are they good for?

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⇒ *e.g. number of meetings between decision makers, violent events, legislative proposals, etc.*

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$\Rightarrow$ *Variables are on the exposure level; related to when (where) the events took place.*

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⇒ *We replace the normal distribution with another probability distribution*

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Formula

The equation the model estimates:

$$E(y_i) \equiv h\lambda_i = h \times exp(\alpha + \beta \times x_i) \quad (1)$$

Estimation of the exposure

What to do with the exposure parameter?

$$E(y_i) \equiv h\lambda_i = h \times exp(\alpha + \beta \times x_i) \quad (2)$$

Two strategies :

- **Offset:** Move it into the equation but constrain parameter: $exp(\alpha + \beta \times x_i + 1 \times log(h_i))$

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$\Rightarrow$ *If the exposure is the same for all units, we set it to 1 and ignore it.*

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$\Rightarrow$ *Make scenarios, predict, knock yourself out*

Dispersion

The main assumption of the Poisson model

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$\Rightarrow$ *The standerd errors will be too small*

Identifying overdispersion

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- Events are related

Adressing overdispersion

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$\Rightarrow$ β *remains the same, standard errors are larger*

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- One producing (at least some) positive counts $\Rightarrow$ *We can model this in two parallel regressions*

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$\Rightarrow$ *Can accomodate under-dispersion too.*

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